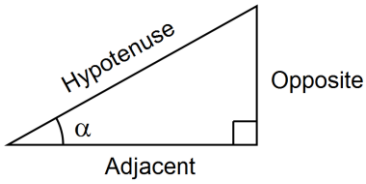
## The Trigonometric Functions

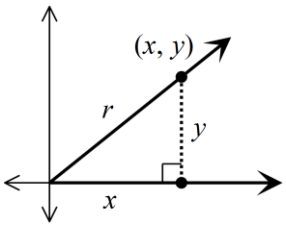
The six trigonometric functions are the sine (sin), cosine (cos), tangent (tan), cosecant (csc), secant (sec), and cotangent (cot) functions. There are several ways to define these functions of trigonometry. One of the most common mnemonic devices is SOH-CAH-TOA.




$$\sin\alpha = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\alpha = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\alpha = \frac{\text{opposite}}{\text{adjacent}}$$

All of these ratios can be written in terms of an angle in the coordinate plane.

If $(x, y)$ is any point other than the origin on the terminal side of an angle $\alpha$ in standard position and $r = \sqrt{x^2 + y^2}$, then



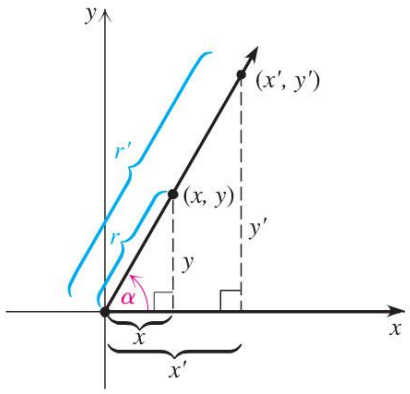

$$\sin\alpha = \frac{\text{opp}}{\text{hyp}} = \frac{y}{r} \qquad \cos\alpha = \frac{\text{adj}}{\text{hyp}} = \frac{x}{r} \qquad \tan\alpha = \frac{\text{opp}}{\text{adj}} = \frac{y}{x}$$

$$\csc\alpha = \frac{\text{hyp}}{\text{opp}} = \frac{r}{y} \qquad \sec\alpha = \frac{\text{hyp}}{\text{adj}} = \frac{r}{x} \qquad \cot\alpha = \frac{\text{adj}}{\text{opp}} = \frac{x}{y}$$

**Reciprocal Identities:** $\csc\alpha = \dfrac{1}{\sin\alpha}$ $\sec\alpha = \dfrac{1}{\cos\alpha}$ $\cot\alpha = \dfrac{1}{\tan\alpha}$

**Examples:**
Find the values of the six trigonometric functions of the angle $\alpha$ in standard position whose terminal side passes through (-2, -4).

To find the values of the trigonometric functions of the "special angles" on the unit circle (multiples of 30° and 45°). We could choose any point on the terminal side of each angle and the same ratios would result because the triangles used to calculate the ratios are similar.




$$\sin\alpha = \frac{y}{r} = \frac{y'}{r'} \qquad \cos\alpha = \frac{x}{r} = \frac{x'}{r'} \qquad \tan\alpha = \frac{y}{x} = \frac{y'}{x'}$$

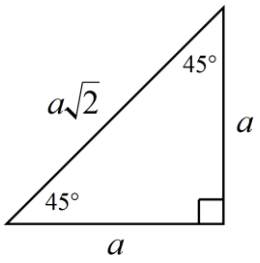
Since $r = 1$ for any point on the unit circle, points on the unit circle are convenient to use for calculating trigonometry ratios.

**On the Unit Circle:**

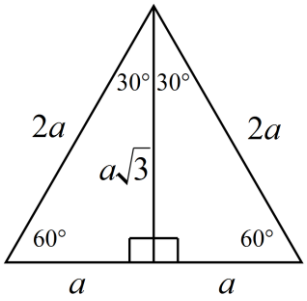
$$\sin\alpha = y \qquad \cos\alpha = x \qquad \tan\alpha = \frac{y}{x}$$

$$\csc\alpha = \frac{1}{y} \qquad \sec\alpha = \frac{1}{x} \qquad \cot\alpha = \frac{x}{y}$$

We can find the use the ratios that exist in special right triangles to calculate the coordinates of points on the unit circle.

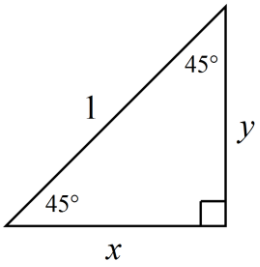


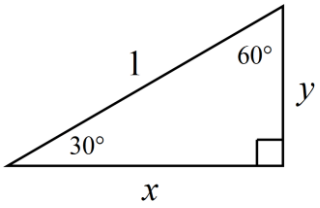

$$\text{hypotenuse} = \text{leg} \cdot \sqrt{2}$$

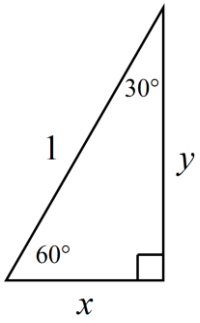



$$\text{hypotenuse} = 2 \cdot \text{short leg}$$
$$\text{long leg} = \text{short leg} \cdot \sqrt{3}$$

Find the values of $x$ and $y$ in the triangles below:




The *signs* of the trigonometric functions depend on the quadrant in which the angle lies and the corresponding signs of $x$ and $y$ (remember $r$ is always positive).

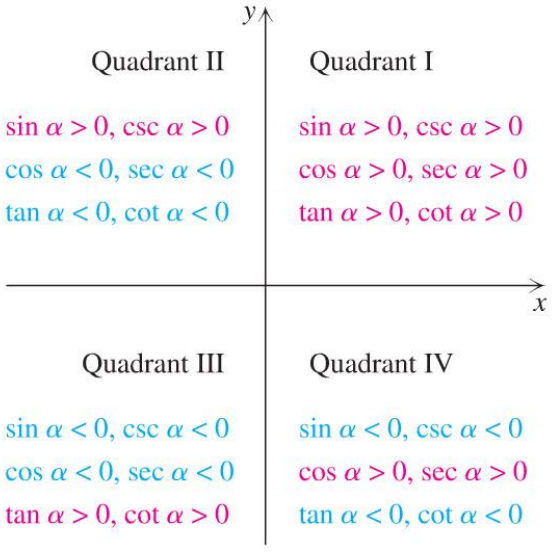
A good mnemonic to remember which functions are positive in each quadrant is "**All S**tudents **T**ake **C**alculus":

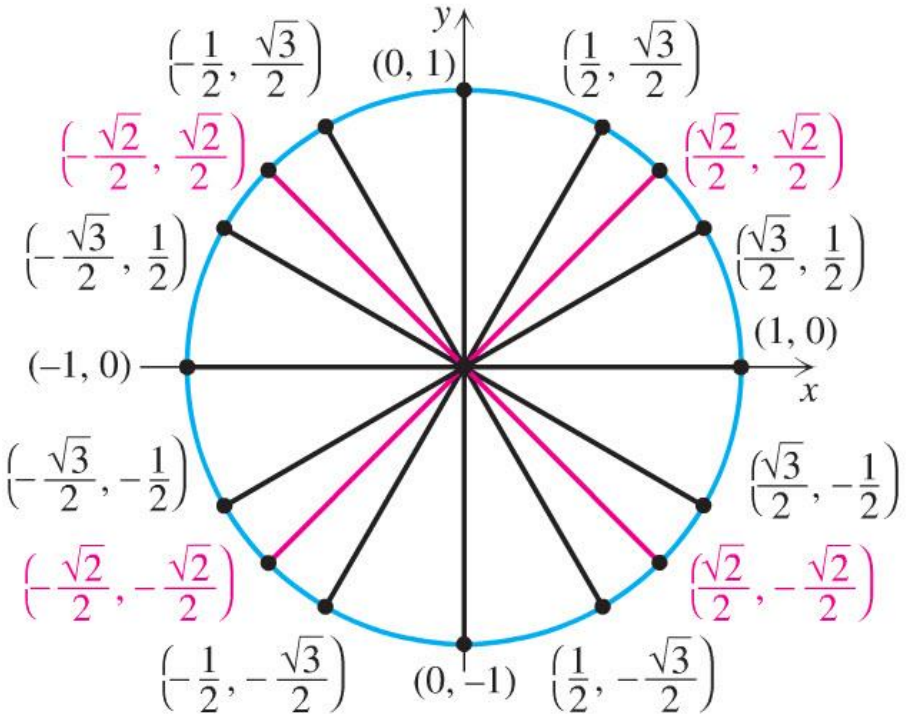
Quadrant I: All of them are positive

Quadrant II: sin and csc are positive

Quadrant III: tan and cot are positive

Quadrant IV: cos and sec are positive






**Examples:** Find the exact values of the following:

1. $\sin 0°$
2. $\cos \pi$
3. $\tan(-\pi/2)$
4. $\csc(-270°)$
5. $\sin(\pi/4)$
6. $\cos(-225°)$
7. $\cot(5\pi/4)$
8. $\sec 315°$
9. $\sin 30°$
10. $\cos(7\pi/6)$
11. $\tan(-\pi/3)$
12. $\csc 150°$
13. $\cot(-240°)$
14. $\sec(-\pi/6)$
15. $\cos(5\pi/3)$
16. $\tan(-150°)$